

# Holographic conductivity of 1+1 dimensional systems in soft wall model

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We study the optical conductivity of 1+1 dimensional systems using soft wall model in the bottom up approach of AdS/CFT (anti-de Sitter/conformal field theory) duality. We find the numerical results for optical conductivity and investigate the system using holographic model in the probe limit. The dependence of conductivity on chemical potential is also investigated. Further, we extend the soft wall model as a ‘no-wall’ model by eliminating the dilaton background and study the response of the system in a simplified approach.

**Keywords:** AdS/CFT duality; soft wall model; no-wall model

## I. INTRODUCTION

The failure of perturbative method and the limitations of traditional non-perturbative approaches makes it difficult to study strongly correlated systems. Lattice field theory[1], is an effective method to study the static equilibrium at high temperature and low density, but requires very high performance computational techniques. However, with the introduction of AdS/CFT correspondence by Maldacena[2] the situation turns favorable. The correspondence relates  $\mathcal{N} = 4$  super Yang-Mills (SYM) conformal field theory in four-dimensions with the Type II  $B$  string theory on  $AdS_5 \times S^5$  spacetime. The generalisation of this conjecture known as gauge/gravity duality or holography and is used to obtain many significant results about the thermodynamical and hydrodynamical aspects of strongly coupled systems using their weakly coupled gravity dual[3–13]. Using the classical gravity solutions in the bulk, we can study holographic theory for the gauged system on the boundary. These observations further leads to the development in understanding of various phenomena like phase transition, Hall effect, Nerst effect and other experimentally observed properties in a strongly correlated condensed matter systems[14–28].

In this work, we study the optical conductivity of a 1+1 dimensional system using its gravity dual BTZ black hole in 2+1 dimensions[29] in the soft wall model[30–32]. This system has been investigated recently[33–40] and the transport properties have been studied both numerically and analytically using charged scalar field coupled with the gauge field in the gravity action. The objective of our work is to calculate the transport properties of the system in much simpler form using soft-wall model[41, 42]. It has been shown in [41] that different dilaton profile corresponds to different condensates for the system. Although, in this present work we have not investigated the phase transition explicitly, but the pattern for the optical conductivity indicates the metallic

phase for 1+1 dimensional system. Using the soft wall model, first we investigate the optical conductivity for the chargeless black hole. Then, the real and imaginary part of the optical conductivity are obtained for different value of the chemical potentials in the ‘no wall’ model by eliminating the dilaton background in front of the Ricci curvature in the action [41, 43].

## Holographic Setup

Let us consider the soft wall model in 2+1 dimensional Einstein-Maxwell system given as,

$$S = \int d^3x \sqrt{-g} e^{-2\phi} \left( \frac{1}{2\kappa^2} (R - 2\Lambda) + \frac{1}{4g^2} F^2 \right) \quad (1)$$

where  $F^2$  is field strength of U(1) gauge field  $A_\mu$ ,  $\kappa^2 = 8\pi G_3 = 1$  ( $G_3$  is the three dimensional Newton’s constant) and  $\Lambda$ (cosmological constant) =  $-2/L^2$  (taking  $L=1$  which is a AdS length scale).

The equations of motion using the above action are given as,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 8\pi G_3 T_{\mu\nu} \quad (2)$$

$$\nabla_\mu e^{-2\phi} F^{\mu\nu} = 0 \quad (3)$$

with  $T_{\mu\nu}$  the energy-momentum tensor. The solution of the equations of motion is a charged black hole system and is given by the following metric ansatz,

$$ds^2 = \frac{1}{z^2} \left( -f(z) dt^2 + dx^2 + \frac{dz^2}{f(z)} \right) \quad (4)$$

where

$$A_t = \mu \ln \frac{z}{z_h}, \quad f(z) = 1 - \frac{z^2}{z_h^2} + \frac{1}{2} \mu^2 z^2 \ln \frac{z}{z_h} \quad (5)$$

and  $\mu$  is the chemical potential. Further we set  $z \rightarrow 0$  as the boundary of  $AdS_3$  and  $z = z_h$  is taken to be the horizon of the black hole with a limit  $z_h \rightarrow 1$  used in

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this paper. The Hawking temperature of the black hole is given by,

$$T = \frac{f'(z)}{4\pi} \Big|_{z \rightarrow z_h} = \frac{4 - \mu^2}{8\pi} \quad (6)$$

The gauge field perturbation used for the calculation is given as

$$A_x(z, t) = \tilde{A}_x(z) e^{-i\omega t} \quad (7)$$

Then, the optical conductivity is obtained using the Ohm's law,

$$\sigma(\omega) = \frac{J^x}{E} = \frac{J^x}{i\omega A_x^0} \quad (8)$$

where  $A_x^0$  act as the source and  $J^x$  is the response for the boundary theory. After scaling the metric perturbations as  $\tilde{h}_{mn} = e^{2\phi} h_{mn}$  we get following equations of motion (at  $\mu = 0$ ).

$$\tilde{A}_x'' + \left( \frac{f'}{f} - 2\phi' \right) \tilde{A}_x' + \frac{\omega^2}{f^2} \tilde{A}_x = 0 \quad (9)$$

Here we have studied two different models for the dilaton profile, Model I ( $\phi = z$ ) and Model-II ( $\phi = z^2$ ) for studying the 1+1 dimensional system in the probe limit. Since it is believed that the dilaton profile is emerging from the condensation of the scalar field, the two different shapes are analogous to the type-II and type-I coherence factors of the holographic superconductor [41].

The numerical results of the optical conductivity using equation (9) has been shown in Fig.1. where the behavior of the optical conductivity matches with the literature [17, 41].

### Optical conductivity for charged black hole in 2+1 dimensions

We study the optical conductivity with chemical potential in the soft wall model. For convenience, in addition to the scaling of metric perturbation, the gauge field has been redefined as,  $\tilde{A}_x = e^\phi A_x$ . This scaling will eliminate the dilaton background and the model works like a 'no-wall' model. Introducing the metric and gauge field perturbations as,

$$g_{mn}(z, t) = h_{mn}(z) e^{-i\omega t} \quad (10)$$

$$A_m(z, t) = \tilde{A}_m(z) e^{-i\omega t} \quad (11)$$

We obtain the equation of motion (in the gauge  $A_r = 0$  and eliminating  $h_{tx}$  using its equations of motion),

$$\begin{aligned} & A_x'' + \left( \frac{f'}{f} + \frac{1}{z} \right) A_x' + \\ & \left( \phi'' - \phi'^2 + \phi' \left( \frac{f'}{f} + \frac{1}{z} \right) - \frac{A_t'^2 z^2 e^\phi}{f} + \frac{\omega^2}{f^2} \right) A_x = 0 \end{aligned} \quad (12)$$

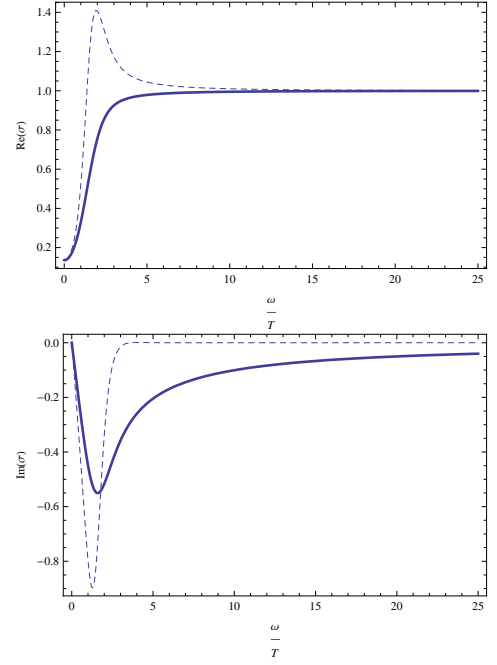


FIG. 1: Frequency response of optical conductivity at  $\mu = 0$  for Model-I ( $\phi = z$ (thick)) and Model-II ( $\phi = z^2$ (dashed)).

In the probe limit of the charged black hole solution, the temporal component of the gauge field at the horizon is given by  $A_t = \mu \ln[\frac{z}{z_h}]$ . Using the ingoing boundary condition,

$$A_x|_{z=1} (1 - z^2)^{-i\omega/2} + \dots \quad (13)$$

Optical conductivity is given by [34–36],

$$\sigma(\omega) = \frac{-(A_x - z A_x' \ln[z])}{z A_x'} \Big|_{z \rightarrow \epsilon} \quad (14)$$

Introducing a small value of chemical potential in the probe limit (taking  $f(z) = 1 - z^2$ ) we study the frequency response of the conductivity in Fig. 2. The presence of Drude peak has been observed for both the models at low frequency indicating the metallic phase of the system.

Further the dependence of conductivity on chemical potential in the probe limit can be verified from Fig.3 and Fig.4. We see the clear shift in the peak along with the suppression of the conductivity as we increase the chemical potential of the system. We also notice that the flow is same for the different chemical potential at higher frequency.

Finally, we consider the charged black hole solution with  $f(z) = 1 - \frac{z^2}{z_h^2} + \frac{1}{2}\mu^2 z^2 \ln \frac{z}{z_h}$  and study the behavior for the real and imaginary part of the optical conductivity at high frequency. It has been noticed earlier that oscillatory behavior indicates the interference effects due to the charge fractionalization [44]. The results has been shown in Fig.5 for both the models at fixed chemical potential.

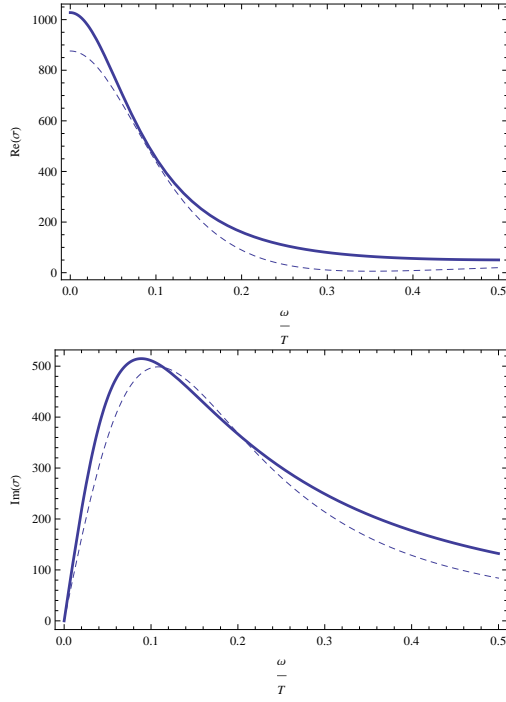


FIG. 2: Frequency response of conductivity for the metallic phase of 1+1 dimensional system at  $\mu = 0.2$ , for Model-I ( $\phi = z$ )(thick) and Model-II( $\phi = z^2$ )(dashed).

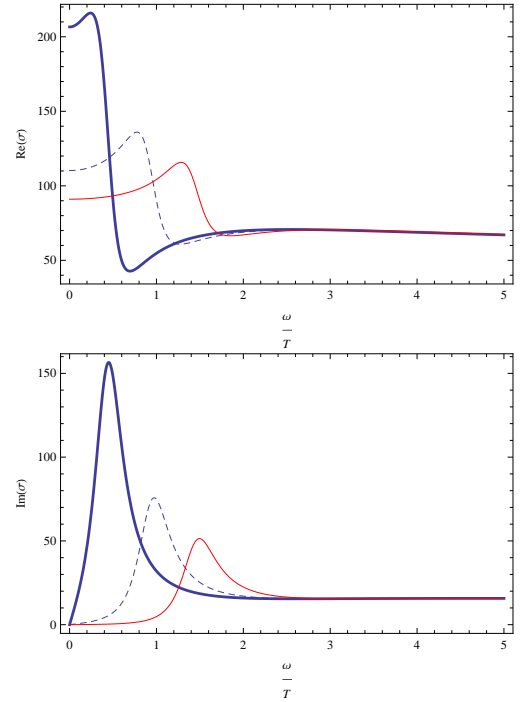


FIG. 4: Frequency response of optical conductivity at  $\mu = 0.5$ (thick), 1(dashed), 1.5(red) for Model-II ( $\phi = z^2$ ).

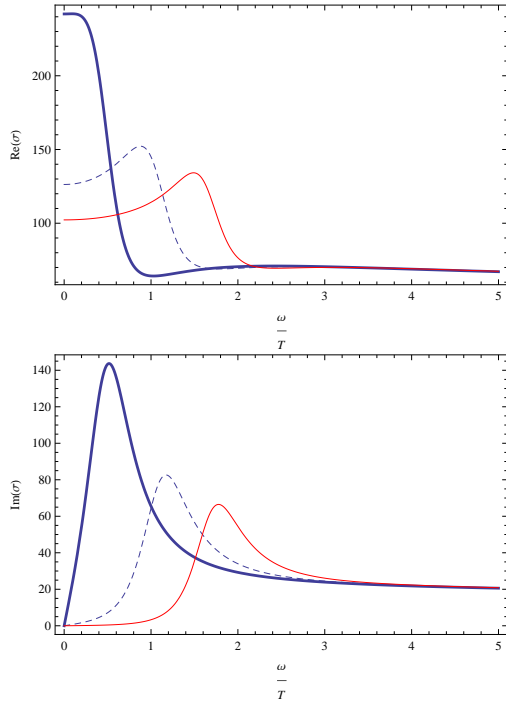


FIG. 3: Frequency response of conductivity at  $\mu = 0.5$ (thick), 1(dashed), 1.5(red) for Model-I ( $\phi = z$ ).

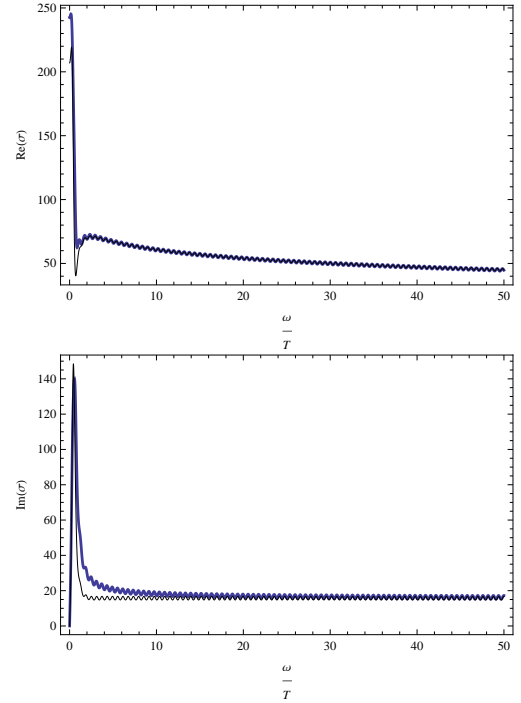


FIG. 5: Frequency response of conductivity at  $\mu = 0.5$ , for Model-I(blue)( $\phi = z$ ) and Model-II(black) ( $\phi = z^2$  )

## Conclusions and Discussions

We have studied the numerical results of optical conductivity for a 1+1 dimensional system in soft wall model using the charged 2+1 dimensional BTZ black hole in the bulk. We have studied two different dilaton profiles to solve the gauge field equation and study the behavior of optical conductivity indicating the Fermi-Luttinger liquid type behavior[34]. We also notice the suppression of conductivity peak as we increase the value of chemical potential of the system. It would be interesting to study the phase structure of the above system with chemical potential in order to decide the existence of superconductivity like phase as indicated in the literature[35].

## Conflict of Interests

The authors declares that there is no conflict of interest regarding the publication of this paper.

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